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METAONTOLOGY DEDS: OPERATIONAL DYNAMIC SYSTEM ON CLASSES

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Abstract
In the present article the parametrical model of dynamic system on classes of objects in structural type COD (evt) [1] formally is under construction, allowing solving an automation problem on function of management with forecasting. The model is under construction on the basis of a coordination principle through a variation of factors of the expenses or security used in models of industrial systems.

Keywords
dynamic system, decision-making, cooridination principle.

Introduction

The decision of a problem of automation of the information technologies connected with function of management in organizational-technological systems, consists in consecutive synthesis *of set of models* of object of management, *set of problems* of the decision-making set on models of object of management and *set of their decisions*, corresponding to management.

Research of the decision of a problem of automation assumes use of the general form of representation of models of dynamics of object of management.

As such form it is offered to use dynamics definition in space of states of the objects, constructed in structural type COD (evt) on the basis of the ontological approach to the analysis of process of modeling [1].

Article is constructed as follows. In Sec. 1 the sense of formal construction of dynamic system on classes is defined. In Sec. 2 construction of such system is carried out on the basis of a coordination principle. In particular, through a variation of factors of the expenses used in models of economic dynamics. In Sec. 3 the industrial interpretation is discussed.

Formal construction of dynamic system and classes

In work the structural type has been constructed:

$$\mathrm{COD\ (evt)} = <\mathrm{Evt},\ \mathrm{A}(\rightarrow)> .$$

Carrier COD (evt) is the space of events with objects which represents set of three of a kind:

$$\mathrm{Evt} = \{\, < ob,\ s,\ t > | ob \in V(ob), s\ S \in (ob), t \in T\},$$

where $V\ (ob)$ there is a set of elementary conceptual objects. One elementary *conceptual object ob* is pair [1]:

$$\mathrm{ob} = ([ob],\ A(ob)),$$

where $[ob]$ – the general name of the objects which are elements-copies of set-class of objects $A(ob)$. Let's consider that the state $s \in S(\mathrm{ob})$ of *object* ob is capacity $|A(ob)|$ of a class *And* (ob) or its some function $f(|A(ob)|)$, e.g.:

$$s = |A(ob)| \text{ or } s = f(|A(ob)|). \tag{1}$$

Then the space of states of object ob will be:

$$\mathrm{S1\ (ob)} = R^1.$$

By definition *event with object ob on an element of its dynamics* is a kind three:

$$\mathrm{evt}(ob) =< ob,\ s,\ t >,$$

where the pair (s, t) is called *as an element of dynamics* of object *ob*. In structural type COD (evt) which support is set Evt, it is considered two basic types dynamics of objects [1]:

– *Elementary time dynamics* or T-dynamics, defined as mapping

$$\partial(\mathrm{ob}) : T \to \mathrm{S1}\ (\mathrm{ob}),$$

where T – the ordered set of the moments of time.

– *Elementary operational dynamics* or S – dynamics, defined as operational correspondence:

$$\sigma^F : \mathrm{evt}(ob_0)^{\mathrm{evt}(ob1)} \to \mathrm{evt}(ob_2)$$

connecting three of events:

$$\mathrm{evt}(ob_0), \quad \mathrm{evt}(ob_1), \quad \mathrm{evt}(ob_2).$$

On the basis of (1) sense *of T-dynamics* consists that capacity of a class $A(ob)$ is defined as function of time T. The sense of S-dynamics is that capacity of a class $A(ob)$ is defined by the operation defined on elements of other classes. Thereby dynamics of conceptual elementary objects represent dynamics of classes corresponding to them. Let's choose as model of time $T = Z$ – set of integers and we will replace a designation *of an element of dynamics* of class A (ob), e.g. pair:

$$(s, k) = (|A(ob)|, k), \quad k \in Z \tag{2}$$

on single symbol – s (k).

We will designate dynamics of class $A(ob)$ through:

$$\partial A(ob) = \{s(k)|k \in Z\}$$

In work [1] it is shown that operational dynamics σ^F is dynamic system which is formally defined as interpretation *Int* [1 (→] a *formal arrow* 1 (→) in space of events with objects:

$$Int\ [1(\to)] = \sigma^F.$$

In this case it is set Evt – set of events with classes. Formal arrow is a symbolical design of a kind:

$$1(\to) =< (s \to);\ dom[^{cen}\to]cod >,$$

where symbols *dom* – the arrow beginning, *cen* – the arrow centre, *cod* – the arrow end are categorial variables, associeted with an internal arrow-correspondence "→" from 1 (→). Hence, the *problem of formal construction of dynamic system* σ^F on classes consists in construction of interpretation of a formal arrow 1 (→) in event space Evt.

By definition, interpretation *Int* [1 (→)] consists in a choice *of values* of variables *dom, cen* and *cod.* The decision of this decision-making problem of in Evt takes the form of assignments of values to these variables:

$$dom = \mathrm{evt}(ob_0), \quad cen = \mathrm{evt}(ob_1), \quad cod\ = \mathrm{evt}(ob_2).$$

Also it is defined at level of classes:

$$A(ob_0), \quad A(ob_1), \quad A(ob_2) \tag{3}$$

and elements of dynamics of these classes. We will name these classes *initial, central and final.*

Coordination T-dynamics of classes

Let's consider classes $A(ob_0)$, $A(ob_2)$. Let dynamics ∂O("→") is set in the form of the known balance equation [2]:

$$s_3(k+1) = s_3(k) + a(k) - b(k), \quad k \in Z, \text{ e.g.:}$$
$$s_3(k) = |O(\ll\!\to\!\gg)| \text{ or } f(|O(\ll\!\to\!\gg)|).$$

Let's define interpretation Int [1 (→)] in Evt as following conformity:

$$\begin{aligned} [\,\to\,] &\Rightarrow s_3(k) \in \partial(\ll\!\to\!\gg), \\ dom &\Rightarrow s_0(k) = a(k) \in \partial(ob_0), \\ cod &\Rightarrow s_2(k) = b(k+1)\partial(ob_2). \end{aligned} \tag{4}$$

To the given interpretation in Evt there corresponds the diagramme of time coordination shown in Fig. 1, which shows that interpretation (4) provides the coordination between elements of dynamics $\partial A(ob_0)$ and $\partial A(ob_2)$ during time moments $t = k$, $t = k + 1$, at which the condition not emptiness of a class $O(\ll\!\to\!\gg)$ is satisfied at any moment $t = k$.

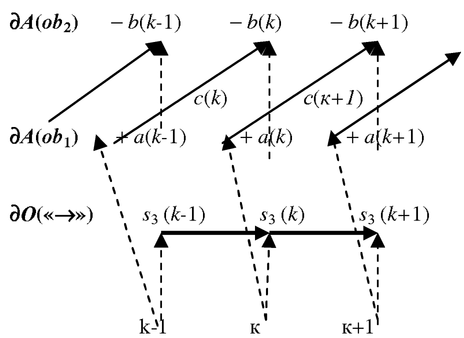

Fig. 1. Diagramme of time coordination.

Really, for any $k \in Z$ at $s_3(0) = 0$ performance of a condition a $(k) = b$ $(k + 1)$ provides:

$$s_3(k) = a(k) > 0.$$

It means that $O(\ll\!\to\!\gg) \subseteq (ob_0)$ and if $A(ob_0) \neq \emptyset$ then $O(\ll\!\to\!\gg) \neq \emptyset$ also, on $\tau(\to) = +1$. Thus, coordination on time for dynamics $\partial A(ob_0)$ and $\partial A(ob_2)$, given by interpretation (4) is defined by conditions

$$\begin{cases} s_3(k+1) = s_3(k) + a(k) - b(k), \quad k \in Z, \\ a(k) = b(k+1), \quad b(k) = a(k-1), \end{cases} \tag{5}$$

which provide *transition* possibility *dom* $\to$ *cod*, e.g. existence of dynamic system σ^F in the form of pair of co-ordinated dynamics:

$$\partial(\ll\to\gg) = (\partial A(ob_0), \partial A(ob_2))$$

satisfying to a condition (5). We name this pair *dynamics of transition*.

Coordination of dynamics of transition with T-dynamics of the central class on states

From definition of an element of dynamics of a class (2) follows that for co-ordinated on time dynamics coordination on states consists in a conformity establishment between the conditions entering into elements of dynamics, carried to some moment of time t. Let $t = k+1$. According to (5), at co-ordinated on time *Int* [1 ($\to$)] in Evt the conformity defining *value of an arrow* takes place:

$$[\to] \Rightarrow s_3(k+1) \in \partial(\ll\to\gg). \tag{6}$$

On the other hand Int [1 ($\to$)] in Evt includes the conformity defining *value of the centre* of an arrow:

$$cen \Rightarrow c(k+1) \in \partial(ob_1).$$

Hence, simultaneously with (7) conformity takes place:

$$[\to] \Rightarrow c(k+1) \in \partial(ob_1). \tag{7}$$

To define coordination on states we will consider a following condition of balance of classes O ($\ll\to\gg$) and

$$\begin{gathered} A(ob_1) \text{ on dynamics } \partial(\ll\to\gg), \\ c(\kappa+1) = h(k+1)s_3(k+1), \end{gathered} \tag{8}$$

where $h(k+1)$ – the characteristic of a class $A(ob_1)$ concerning a class O ($\ll\to\gg$), corresponding to transition dynamics. Substitution in (9) expressions for s_3 $(k+1)$ from (6) leads to following conditions

$$\begin{cases} c(\kappa+1) = h(k+1)(s_3(k) + a(k) - b(k)), \\ a(k) = b(k+1), \qquad b(k) = a(k-1), \end{cases} \tag{9}$$

defining coordination of elements of three T-dynamics $\partial A(ob_0)$, $\partial A(ob_1)$ and $\partial A(ob_2)$ concerning T-dynamics of a class O ($\ll\to\gg$). It is easy to see, (10) are conditions of the coordination of *dynamics of transition* $\partial(\ll\to\gg) = (\partial A(ob_0),\ \partial A(ob_2))$ with dynamics of the centre $\partial A(ob_1)$ on the states, providing existence of dynamic system σ^F as co-ordinated interpretation of T-dynamics in *Int* [1 ($\to$)] in Evt.

As criterion of dynamics coordination we will consider concept of attainability. Let's say that T-dynamics $\partial(\ll\to\gg)$ and $\partial A(ob_1)$ are co-ordinated on states at the moment of time $t = k+1$ if the system σ^F is *attainable* at the moment of time $t = k+1$ e.g. if the condition:

$$cod = b(k+1) > 0 \tag{10}$$

is satisfied. It is easy to see that as $a\ (k) = b\ (k+1)$ owing to the first condition in (10) so the problem about attainability of system σ^F can be formulated as the following problem of decision-making $Z^0 < a^0,\ h^0 >$ with two criteria:

$$\begin{gathered} \max a(k)/a^0, \ \ 0 < a(k)a \leq^0 = b(k+1), \\ \max h(k+1)/h^0, \ 0 < h(k+1) \leq h^0, \\ c(\kappa+1) = h(k+1)(s_3(k) + a(k) - b(k)), \end{gathered}$$

with set of admissible decisions:

$$\begin{gathered} \{ < a(k), h(k+1) > |c(\kappa+1) = \\ = h(k+1)(s_3(k) + a(k) - b(k))\}. \end{gathered}$$

This problem is a problem on *simultaneously reached maximum* and it's any admissible decision there is a pareto-optimum. The best pareto-optimum decision of this problem is the *free point* (a^0, h^0).

In article [3] it was shown that always it is necessary to search for its optimum decision through a finding of the optimum decision of problem $Z^1 < a^0,\ h^0 >$:

$$\begin{gathered} \max[\min\{a(k)/a^0, h(k+1)/h^0\}], \\ c(\kappa+1) = h(k+1)(s_3(k) + a(k) - b(k)), \end{gathered}$$

with the same set of admissible decisions. Interrelation of problems $Z^0 < a^0,\ h^0 >$ and $Z^1 < a^0,\ h^0 >$ consists that if the free point (a^0, h^0) is the admissible decision it is the optimum decision of both these problems.

Discussion

The dynamic system σ^F as co-ordinated interpretation *Int* [1($\to$)] in Evt is mathematical model of some *industrial system* σ, constructed in structural type COD(evt). The basis of this model is made by a condition of material balance (9) on the balance equation [2]:

$$c(\kappa+1) = h(k+1)(s_3(k) + a(k) - b(k)) \tag{11}$$

which defines *industrial system* σ as *dynamic whole* [1] given in the structural form of set of objects (products) changing in time. The variables of Eq. (12) are following parametres of *industrial systems* σ:

- s_3 (k) – quantity of production which is in process of manufacture in *industrial systems* σ on a interval of time $\tau(\to) = +1$;

- $a\ (k)$ – *input* of *industrial systems* σ at the moment of time $t = k$ which is equal to $b\ (k + 1)$ – planned *output* of system σ at the moment of time $t = k + 1$;
- $b\ (k)$ – *output* of *industrial systems* σ at the moment of time $t = k$;
- $c(\kappa + 1)$ – quantity of resources of *industrial systems* σ, used in the course of manufacture on an interval of time $\tau(\rightarrow) = +1$;
- $h(k + 1)$ – the factor of security in industrial systems σ which can be considered by resources of manufacture of a unit of production as factor of expenses of resources (or local interpretation of factors of the expenses from Leonie's model) in the given system balance equation:

$$s_3(k + 1) = s_3(k) + a(k) - b(k), \quad k \in Z.$$

The balance equation entering in condition (12) is often used for management with forecasting [2].

It is easy to see that according to the first criterion the decision of problem $Z^1 < a^0,\ h^0 >$ is value of planned output $a\ (k)$ of *industrial systems* σ at the moment of time $t = k + 1$ (*input* of *industrial systems* σ at the moment of time $t = k$) depending on predicted requirement for this production set in size $b(k + 1)$. According to the second criterion received value $a\ (k)$ should be co-ordinated with $h\ (k + 1)$ – *factor of security* of *industrial system* σ.

Hence, problem $Z^1 < a^0,\ h^0 >$ represents function of co-ordinated management with forecasting, where parametre $c(\kappa + 1)$ represents possibility of development of resourńes of *industrial systems* σ in time.

Example

In Introduction it was noted that *automation* of the information technologies connected with function of management combines three basic functions: *modeling*, *decision making* and *set of decisions* corresponding to management. The following example shows how this problem can be solved using structural type COD(evt) from metaontology DEDS [1].

From the logical point of view *modeling* based on metaontology DEDS is the interpretation of dynamic system σ^F defined through description of parameters of the problem of decision-making $Z^0 < a^0,\ h^0 >$ in the lexicon of specific manufacturing field.

Consider the higher education industry as a field of interpretation then the dynamic system σ^F or *industrial systems* σ is *a institution of higher education.* In this case the parameters of the problem of decision-making $Z^0 < a^0,\ h^0 >$ are defined as follows:

- $s_3\ (k)$ – *number of students* of institution of *higher education* σ at the moment of time $t = k$;
- $a\ (k)$ – *number of students admitted to the institution* of higher education σ at the moment of time $t = k$ which in the best case is equal to $b\ (k+1)$ – demand *or need of graduates* of institution of higher education σ at the moment of time $t = k + 1$;
- $b\ (k+1)$ – demand *or need of graduates* of institution of higher education σ at the moment of time $t = k + 1$;
- $b\ (k)$ – *number of students graduating from* institution of higher education σ at the moment of time $t = k$;
- $c(\kappa + 1)$ – classroom foundation of institution of higher education σ, used in the course of manufacture on an interval of time $\tau(\rightarrow) = [k, k + 1]$;
- $h\ (k + 1)$ – *factor of security* or the availability of training area of institution of higher education σ which can be considered to be the resources of learning process needed for one student during an interval of time $\tau(\rightarrow) = +1$.

Solution *of decision-making problem* $Z^0 < a^0,\ h^0 >$ at the moment of time $t = k$ is the value of $a\ (k)$. The following table contains numerical solutions of $Z^0 < a^0,\ h^0 >$ for the case when institution of higher education σ produce specialists in 5 specialties S_j, $j = 1, 2, 3, 4, 5$.

Table 1
Numerical solutions of $Z^0 < a^0, h^0 >$.

S_j	$a^0 = b(k+1)$	$s_3(k)$	$b(k)$	$a(k)$ $h^0 = h(k)$	$a(k)$ $h^0 = h^N$
S_1	175	809	146	168	150
S_2	50	226	39	48	43
S_3	50	243	42	48	43
S_4	75	375	60	72	66
S_5	150	870	134	144	132

The values of parameters $s_3\ (k)$, $a\ (k)$, $b\ (k)$ and $b\ (k + 1)$ relate to each specialty S_j, $j = 1, 2, 3, 4, 5$.

Solutions were obtained for two possible values of parameter h^0:

- $h^0 = h(k) = 8.1\ \mathrm{m}^2$ – *current value* of availability of training area of institution of higher education σ at the moment of time $t = k$;
- $h^0 = h^N = 10.0\ \mathrm{m}^2$ – *normative value* of availability of training area of institution of higher education σ.

The value of classroom foundation was chosen to be:

$$c(\kappa + 1) = c(\kappa) = 30214\ \mathrm{m}^2.$$

The problem of decision-making $Z^0 < a^0,\ h^0 >$ defines $a(k)$ as a function of parameters $(a^0,\ h^0)$, which defines the *set of possible decisions* of which must be chosen value of $a(k)$ – possible solution at the moment of time $t = k$. The possible solution corresponds to the management with forecasting if at the moment of time $t = k$ the parameter $a^0 = b(k + T)$ − planned or forecasted demand *for* specialists produced by institution of higher education σ is given as a function of time parameter $T = \{0, 1, 2, \dots, K\}$.

The decision making problem $Z^0 < a^0,\ h^0 >$ must be solved at each moment of time $t = k$. The table shows that if at the moment of time $t = k$ the planned or forecasted demand *for* specialists S_1 is equal to 175 and the value $h^0 = h(k)) = 8.1$ m^2 is chosen to determine the value of $a(k) \leq a^0 \leq b$ $(k + 1)$ then the decision will be $a * (k) = 168$ but if the value of $h^0 = h^N = 10.0$ m^{2}the decision will be $a * (k) = 150$.

So it is possible to say that the solutions of decision-making problem $Z^0 < a^0,\ h^0 >$ correspond to the management implementing the principle of tracking system when solution $a * (k)$ tracks the forecasted demand $b(k + 1)$.

Conclusions

The dynamic system σ^F on classes in structural type COD(evt) is formally constructed. At system construction the *coordination principle has been used*. The received results can be used for the solution of a problem of automation of information technology of management in the organizational-technological systems, realised in the form of consecutive synthesis *of model* of *system based on coordination principle* the *problems of decision-making* set on model of system and *set of their decisions*, corresponding to management with forecasting based on coordination principle.

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